Math 115

Fall 2018

Lecture 22

$? \; a^2 + b^2 = c^2 \; ?$

$y = mx + b \quad ? \quad d = rt$

Factor out the GCF:

① $36x^3 + 24x = \boxed{12x(3x^2 + 2)}$

② $100x^5 - 16x^3 = \boxed{4x^3(25x^2 - 4)}$

↳ Can be factored more.

③ $x(x + 2y) - (x + 2y)$

$= \boxed{(x + 2y)(x - 1)}$

Factor by grouping:

① $x^2 + 2x + xy + 2y$

$= x(x+2) + y(x+2) = \boxed{(x+2)(x+y)}$

② $4x^3 + 3x^2 + 4x + 3$

$= x^2(4x+3) + 1(4x+3) = \boxed{(4x+3)(x^2+1)}$

③ $x^5 + 2x^5y - 6y - 3$

$= x^5(1+2y) - 3(2y+1) = \boxed{(2y+1)(x^5-3)}$

Find the missing factor:

① $x^2 + 11x + 30 = (x+5)(x+6)$ (5x, 6x)

② $x^2 - x - 42 = (x-7)(x+6)$ (-7x, 6x)

③ $x^3 + 6x^2 - 27x = (x-3)(x^2+9x) = \boxed{x(x-3)(x+9)}$ ($-3x^2$, $9x^2$)

Factor Completely:

① $x^2 + 8x + 15$

P = 15
S = 8
15
3 & 5

$= x^2 + 3x + 5x + 15$

$= x(x+3) + 5(x+3)$

$= \boxed{(x+3)(x+5)}$

② $x^2 - 3x - 39$

P = -39
S = -3
-39

1, -39
3, -13

Prime

③ $x^2 - 11x + 24$

P = 24
S = -11
24
-8 & -3

$= x^2 - 8x - 3x + 24$

$= x(x-8) - 3(x-8)$

$= \boxed{(x-8)(x-3)}$

④ $y^2 + 3y - 54$

P = -54
S = 3
-54
9 & -6

$= y^2 + 9y - 6y - 54$

$= y(y+9) - 6(y+9)$

$= \boxed{(y+9)(y-6)}$

Factor out the GCF first, then factor the rest:

① $x^3y + x^2y - 6xy = xy(x^2 + x - 6)$

$= xy(x + 3)(x - 2)$

② $(3x-2)x^2 - 13(3x-2)x + 40(3x-2)$

$= (3x-2)(x^2 - 13x + 40)$

$= \boxed{(3x-2)(x-5)(x-8)}$

1, 40
2, 20
4, 10
5, 8

Factor completely:

① $2x^2 + 7x + 6$

$P = 12$
$S = 7$
$3 \& 4$

$= 2x^2 + 3x + 4x + 6$

$= x(2x+3) + 2(2x+3)$

$= \boxed{(2x+3)(x+2)}$

② $12x^2 - 2 - 5x$

$= 12x^2 - 5x - 2$

$P = -24$
$S = -5$
$3 \& -8$

1, -24
2, -12
3, -8
4, -6

$= 12x^2 + 3x - 8x - 2$

$= 3x(4x+1) - 2(4x+1)$

$= \boxed{(4x+1)(3x-2)}$

Factor Completely:

$8x^3 + 5x - 14x^2$

Hint: GCF ✓ & order ✓

$= 8x^3 - 14x^2 + 5x$

$= x(8x^2 - 14x + 5)$

$P = 40$ ✓

$S = -14$ ✓

-10 & -4

40

$-1, -40$

$-2, -20$

$-4, -10$

$-5, -8$

$8x^2 - 10x - 4x + 5$

$= 2x(4x-5) - 1(4x-5)$

$x(4x-5)(2x-1)$

Factor Completely: $6x^2 - 11xy + 3y^2$

$P = 18$

$S = -11$

$-1, -18$

$-2, -9$

$-3, -6$

18

$= 6x^2 - 2xy - 9xy + 3y^2$

$= 2x(3x-y) - 3y(3x-y)$

$= (3x-y)(2x-3y)$

Special Factoring with Binomials:

① Sum of two squares: $A^2 + B^2$

Prime

② Difference of two squares: $A^2 - B^2$

$$A^2 - B^2 = (A + B)(A - B)$$

ex: $x^2 + 100 = x^2 + 10^2$ **Prime**

ex: $x^2 - 49 = x^2 - 7^2 = \boxed{(x+7)(x-7)}$

ex: $4x^2 + 25 = (2x)^2 + 5^2$ **Prime**

ex: $25x^2 - 36y^2 = (5x)^2 - (6y)^2$

$= (5x + 6y)(5x - 6y)$

ex: $100x^3 - 49x$

Hint: GCF first

$= x(100x^2 - 49)$

$= x\left[(10x)^2 - (7)^2\right] = \boxed{x(10x+7)(10x-7)}$

Factor: $x^2(x+6) - 36(x+6)$ Hint: GCF

$= (x+6)(x^2-36)$

$= (x+6)(x^2-6^2)$

$= (x+6)(x+6)(x-6)$

$= \boxed{(x+6)^2(x-6)}$

Special Factoring with Binomials:

① Sum of two squares: $A^2 + B^2$

Prime

② Difference of two squares: $A^2 - B^2$

$A^2 - B^2 = (A+B)(A-B)$

③ Sum of two cubes: $A^3 + B^3$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

$x^3 + 27 = x^3 + 3^3 = \boxed{(x+3)(x^2 - 3x + 9)}$

Factor: $8x^3 + 125 =$

$(2x)^3 + (5)^3 = (2x+5)(4x^2 - 10x + 25)$

$27x^3 + 1000y^3 =$

$(3x)^3 + (10y)^3 = (3x+10y)(9x^2 - 30xy + 100y^2)$

$64x^3 + 125y^3 = (4x)^3 + (5y)^3$

$= (4x + 5y)(16x^2 - 20xy + 25y^2)$

Special Factoring with Binomials:

① Sum of two squares: $A^2 + B^2$

Prime

② Difference of two Squares: $A^2 - B^2$

$A^2 - B^2 = (A + B)(A - B)$

③ Sum of two Cubes: $A^3 + B^3$

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

④ Difference of Two Cubes: $A^3 - B^3$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

Factor $x^3 - 1000$

$= x^3 - 10^3$

$= (x - 10)(x^2 + 10x + 100)$

10^2

Factor $2x^3 - 16y^3$ Hint: GCF

$= 2(x^3 - 8y^3)$

$= 2[x^3 - (2y)^3] = 2(x-2y)(x^2 + 2xy + 4y^2)$

Factor Completely: Hint: GCF

$4x^5 - 500x^2y^3$

$= 4x^2(x^3 - 125y^3)$

$= 4x^2[(x)^3 - (5y)^3]$

$= 4x^2(x - 5y)(x^2 + 5xy + 25y^2)$

Factor Completely

$x^3(x^2-9)-27(x^2-9)$

$=(x^2-9)(x^3-27)$

$=(x+3)(x-3)(x-3)(x^2+3x+9)$

$=\boxed{(x+3)(x-3)^2(x^2+3x+9)}$

Factor: $81x^2-36$

Always do GCF first

$=9(9x^2-4)$

$=9\left[(3x)^2-(2)^2\right]$

$=\boxed{9(3x+2)(3x-2)}$

Factor Completely:

First GCF

$250x^4y - 432xy^7$

$= 2xy(125x^3 - 216y^6)$

$= 2xy[(5x)^3 - (6y^2)^3]$

$= 2xy(5x - 6y^2)(25x^2 + 30xy^2 + 36y^4)$

Special Factoring with Trinomials:

$A^2 + 2AB + B^2 = (A + B)^2$

$x^2 + 20x + 100 = (x + 10)^2$

$2 \cdot x \cdot 10$

$25x^2 + 60x + 36 = (5x + 6)^2$

$2 \cdot 5x \cdot 6$

$100x^2 + 220xy + 121y^2$

$= (10x + 11y)^2$

$2 \cdot 10x \cdot 11y$

$64x^4 + 80x^2y^3 + 25y^6$

$= (8x^2 + 5y^3)^2$ ✓

Special Factoring with Trinomials:

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Factor $x^2 - 14x + 49 = (x - 7)^2$

Factor $x^3 - 50x^2 + 625x =$

$x(x^2 - 50x + 625) =$

$x(x - 25)^2$

Factor Completely:

$$50x^3y - 40x^2y^2 + 8xy^3$$

$$= 2xy\left[25x^2 - 20xy + 4y^2\right]$$

$$= 2xy(5x - 2y)^2$$

$2 \cdot 5x \cdot 2y$

Factor $x^6 - 64$ Completely

Start with difference of two squares.

$$x^6 - 64 = (x^3)^2 - (8)^2$$

$$= (x^3 + 8)(x^3 - 8)$$

$$= (x^3 + 2^3)(x^3 - 2^3)$$

$$= (x+2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4)$$

$$= \boxed{(x+2)(x-2)(x^2 - 2x + 4)(x^2 + 2x + 4)}$$